Shot noise in the half-filled Landau level

Felix von Oppen
Department of Condensed Matter Physics, Weizmann Institute of Science, 76100 Rehovot, Israel
(February 1, 2008)

Shot noise in the half-filled Landau level is studied within the composite-fermion picture, focusing on the diffusive regime. The composite fermions are assumed to form a Fermi liquid with nontrivial Fermi-liquid parameters. The Boltzmann-Langevin equation for this system is derived, taking proper account of fluctuations in both the Chern-Simons and the physical electric and magnetic fields. To leading order in $\max\{eV,T\}/E_F$, the noise properties of composite fermions are found to equal those of semiclassical electrons in the external magnetic field. Non-equilibrium fluctuations in the Hall voltage are dominated by fluctuations in the Chern-Simons electric field, reflecting the finite Hall resistance of the system. The low-frequency noise power is derived in detail for the Corbino-disc geometry and turns out to be unaffected both by nontrivial Fermi-liquid parameters and by the magnetic field. The formalism is also applied to compute thermal density-density and current-current correlators at finite frequency and wavevector.

PACS numbers: 73.40.Hm,72.70.+m,73.23.-b

I. INTRODUCTION

The composite-fermion approach has had considerable success in describing the physics of the two-dimensional electron gas in high magnetic fields.[1,2,3] In this approach, one transforms the problem to a new set of fermions which consist of an even number of fictitious magnetic flux quanta attached to each electron.[2,4] These composite fermions (CF) interact not only by the Coulomb interaction, but also via a fictitious gauge field, the so-called Chern-Simons field. For the compressible states at even-denominator fractions, and in particular for filling factor $\nu = 1/2$, this leads to a mean-field picture of non-interacting composite fermions in zero magnetic field. The effective magnetic field experienced by the composite fermions vanishes because the external field is canceled by the magnetic field associated with the attached flux tubes. Away from half filling, the magnetic-field cancelation is not complete. The principal odd-denominator fractional quantum-Hall states of the original electrons can then be understood as integer quantum-Hall states of composite fermions. It has been argued that this Fermi-liquid picture remains valid even when including corrections to mean-field theory.[2,5,6,7]

The purpose of the present paper is to study non-equilibrium (shot) noise in the half-filled Landau level, focusing on the diffusive regime. Despite the Fermi-liquid nature of composite fermions, a theory of current noise in the half-filled Landau level must take into account two features not present in diffusive conductors at zero magnetic field. First, composite fermions are not only coupled to the physical electric and magnetic fields but also to the Chern-Simons fields originating from the flux lines "attached to the electrons." These fluctuations in the Chern-Simons fields are in turn related to density and current fluctuations, thus requiring a self-consistent solution. Second, corrections to mean-field theory cause strong quasiparticle interactions between the composite fermions.[2,5,6,7] The approach developed in this paper allows one to include both of these features.

Shot noise in mesoscopic systems in zero magnetic field has been extensively investigated in the last few years, both theoretically[8] and experimentally.[9,10] It was found that Fermi correlations of the carriers lead to a suppression of the shot-noise power below its classical (Poisson) value $S_{\rm Poisson} = 2eI$ with I the average current flowing through the device.[11] For metallic samples, shot noise depends on a variety of length scales. For samples shorter than the electron-electron scattering length $L_{\rm e-e}$, it was found that there is a universal reduction factor 1/3 of the shot noise power, $S = (1/3)S_{\rm Poisson}$.[12,13,14] The reduction factor changes for samples larger than $L_{\rm e-e}$ but smaller than the electron-phonon length $L_{\rm e-ph}$, for which one finds $S = (\sqrt{3}/4)S_{\rm Poisson}$.[15,16] (Note that in metals typically $L_{\rm e-e} \ll L_{\rm e-ph}$ at sufficiently low temperatures.) Shot noise vanishes for samples larger than $L_{\rm e-ph}$.[15,16]

Both novel ingredients at $\nu = 1/2$, namely the coupling to Chern-Simons fields and the strong quasiparticle interactions, are naturally incorporated into the Boltzmann-Langevin approach to shot noise. In this approach, one starts from the kinetic equation for the phase-space distribution function. The average distribution function, determining the time-averaged current, is governed by the Boltzmann equation. In the diffusive regime, fluctuations in the distribution function – and hence in the current – arise primarily from the statistical nature of the impurity scattering. Making the standard assumption underlying the kinetic equation that subsequent impurity-scattering events are statistically independent, one can derive a Boltzmann-Langevin equation which governs the fluctuations in the distribution function.[17] Here, we show how such a Boltzmann-Langevin equation can be derived for composite fermions, including both the Chern-Simons fields and the quasiparticle interactions.

In this paper we consider shot noise at $\nu = 1/2$ in the two regimes $\ell_{\rm tr} \ll L \ll L_{\rm cf-cf}$ and $\ell_{\rm tr} \ll L_{\rm cf-cf} \ll L \ll$

$L_{\rm cf-ph}$. Here $\ell_{\rm tr}$ denotes the transport mean free path for impurity scattering, $L_{\rm cf-cf}$ denotes the mean free path for CF-CF scattering, $L_{\rm cf-ph}$ the mean free path for CF-phonon scattering, and L is the sample size. While the length scales $L_{\rm cf-cf}$ and $L_{\rm cf-ph}$ have not been studied in detail for composite fermions, one expects that, by analogy with electrons, $L_{\rm cf-cf} \ll L_{\rm cf-ph}$ at sufficiently low temperatures.[18] We make no further assumptions about these length scales. Instead, they enter into our calculations as empirical parameters and we suggest that shot noise may be used to measure them experimentally.

The paper is organized as follows. In section II we introduce the relevant kinetic equations, both for the average distribution function (II A) and for the fluctuations of the distribution function (II B). There we also derive the consequences for the fluctuations in the current density from the Boltzmann-Langevin equation and compare the results to those for classical diffusive electrons in a magnetic field. This result is applied in section III to compute the low-frequency shot noise for Corbino discs. The thermal density and current correlators at finite frequency and momentum are derived from the Boltzmann-Langevin approach in section IV. Finally, we summarize and conclude in section V. Details of some calculations are presented in two appendices.

II. KINETIC EQUATIONS

A. Boltzmann equation

We will discuss the current and density fluctuations at $\nu = 1/2$ within the framework of the kinetic equation. The starting point is the Fermi-liquid theory of the half-filled Landau level in terms of composite fermions. For completeness and for fixing notation, we start with a brief review of the Boltzmann equation for the average composite-fermion distribution function $n_{\mathbf{p}}(\mathbf{r}, t)$. Since the physical magnetic field is exactly compensated by the Chern-Simons magnetic field at half filling, the Boltzmann equation involves only electric fields. Including arbitrary quasiparticle interactions, the Boltzmann equation linearized in the applied field $\mathbf{E}^{\rm ext}$ is,[19]

$$\frac{\partial}{\partial t}\delta n_{\mathbf{p}} + (\mathbf{v_p}\nabla_{\mathbf{r}})\delta\tilde{n}_{\mathbf{p}} + e(\mathbf{E}+\mathbf{E}^{\rm CS})\nabla_{\mathbf{p}} n^0_{\mathbf{p}} - S_{\mathbf{p}}\{n_{\mathbf{p}}\} = 0. \quad (1)$$

Here $n^0_{\mathbf{p}}$ denotes the equilibrium distribution function, $\delta n_{\mathbf{p}} = n_{\mathbf{p}} - \theta(\mu - \epsilon_{\mathbf{p}})$ the deviations from the ground-state distribution function, and $\delta\tilde{n}_{\mathbf{p}} = n_{\mathbf{p}} - \theta(\mu - \tilde{\epsilon}_{\mathbf{p}})$ the deviations from the local ground state, defined in terms of the local composite-fermion energies $\tilde{\epsilon}_{\mathbf{p}} = \epsilon_{\mathbf{p}} + \sum_{\mathbf{p}'} f_{\mathbf{pp}'}\delta n_{\mathbf{p}'}$ (with $f_{\mathbf{pp}'}$ the Landau function). The charge density $\delta\rho = (e/\Omega)\sum_{\mathbf{p}} \delta n_{\mathbf{p}}$ and current $\mathbf{j} = (e/\Omega)\sum_{\mathbf{p}} \mathbf{v_p}\delta\tilde{n}_{\mathbf{p}}$ are expressed in terms of $n_{\mathbf{p}}$ in the usual way (Ω is the volume of the sample). The physical electric field $\mathbf{E} = \mathbf{E}^{\rm ext} + \mathbf{E}^{\rm ind}$ includes the induced field $\mathbf{E}^{\rm ind}$.

The composite-fermion motion is affected by the physical electric field $\mathbf{E} = -\nabla_{\mathbf{r}}(\phi^{\rm ext} + \phi^{\rm ind})$ with

$$\phi^{\rm ind}(\mathbf{r}) = \int d\mathbf{r}' \frac{\delta\rho(\mathbf{r}')}{\epsilon|\mathbf{r}-\mathbf{r}'|}, \quad (2)$$

(ϵ denotes the dielectric constant) and the Chern-Simons electric field

$$\mathbf{E}^{\rm CS} = \frac{2h}{e^2}(\hat{\mathbf{z}} \times \mathbf{j}), \quad (3)$$

originating from the flux lines moving with the composite fermions ($\hat{\mathbf{z}}$ denotes the unit vector perpendicular to the sample). Both fields are given in terms of the distribution function and need to be determined self consistently.

The velocity entering the Boltzmann equation is related to the momentum via the effective mass, $\mathbf{p} = m^* v_{\mathbf{p}}$. The effective mass diverges due to interactions with fluctuations in the Chern-Simons field.[2] However, there are also singular contributions to the Landau function $f_{\mathbf{pp}'}$ and it has been argued[6,7] that there is a cancellation of divergent terms when calculating response functions. As a result, the effective mass m^* and the Landau function entering the Boltzmann equation can be taken as nonsingular.[6,7] This effective mass is expected to be finite, but larger than the electron band mass. We will not assume any particular form for the Landau function, beyond its being nonsingular, because our final results will turn out to be independent of it.

Composite fermions scatter from impurities, other composite fermions, and phonons with associated collision integrals

$$S_{\mathbf{p}}\{n_{\mathbf{p}}\} = S^{\rm imp}_{\mathbf{p}}\{n_{\mathbf{p}}\} + S^{\rm cf-cf}_{\mathbf{p}}\{n_{\mathbf{p}}\} + S^{\rm cf-ph}_{\mathbf{p}}\{n_{\mathbf{p}}\}. \quad (4)$$

and scattering lengths $\ell_{\rm tr}$, $L_{\rm cf-cf}$, and $L_{\rm cf-ph}$. We assume that the dominant scattering mechanism is due to impurities,

$$S^{\rm imp}_{\mathbf{p}}\{n_{\mathbf{p}}\} = \sum_{\mathbf{p}'} W_{\mathbf{pp}'}\left\{n_{\mathbf{p}'}(1-n_{\mathbf{p}}) - n_{\mathbf{p}}(1-n_{\mathbf{p}'})\right\}. \quad (5)$$

In the relaxation-time approximation employed here this becomes

$$S^{\rm imp}_{\mathbf{p}}\{n_{\mathbf{p}}\} = \frac{1}{\tau_{\rm tr}}\left\{\delta\tilde{n}_{\mathbf{p}} - \int \frac{d\theta_{\mathbf{p}}}{2\pi}\delta\tilde{n}_{\mathbf{p}}\right\}, \quad (6)$$

with $\theta_{\mathbf{p}}$ the direction of $\mathbf{p}$ and $\tau_{\rm tr}$ the transport mean free time.[20] For samples longer than $L_{\rm cf-cf}$, the scattering of composite fermions on one another also needs to be taken into account. This scattering mechanism leads to a Fermi-Dirac distribution function with spatially varying chemical potential and temperature. For samples which are longer than $L_{\rm cf-ph}$, the composite-fermion temperature becomes constant throughout the sample and coincides with the phonon temperature.

The Boltzmann equation (1) is the same as that for electrons in an electric field $\mathcal{E} = \mathbf{E} + \mathbf{E}^{\rm CS}$ so that for

translation-invariant situations $\mathbf{j} = \sigma_{\rm CF}\mathcal{E}$, where the composite-fermion conductivity $\sigma_{\rm CF} = e^2 N(0) D$ is given by the Einstein relation. ($N(0)$ is the density of states at the Fermi energy and D the diffusion constant.) The physical conductivity is defined as the response to the physical electric field $\mathbf{E}$. Eliminating the Chern-Simons electric field (3) from this equation, one reads off the physical resistivity tensor

$$\hat{\rho} = \begin{pmatrix} 1/\sigma_{\rm CF} & 2h/e^2 \\ -2h/e^2 & 1/\sigma_{\rm CF} \end{pmatrix}. \tag{7}$$

B. Boltzmann-Langevin equation

In deriving the Boltzmann equation, it is assumed that subsequent collisions of quasiparticles with impurities or other quasiparticles are statistically independent. Hence, these scattering mechanisms are Poisson processes with fluctuations equal to the average number of scattering events. These fluctuations in the scattering rates cause fluctuations of the distribution function $\Delta n_{\mathbf{p}}$ around its average. A kinetic equation for $\Delta n_{\mathbf{p}}$ is readily derived following Kogan and Shul'man.[17] The statistical fluctuations in the scattering rates enter the resulting Boltzmann-Langevin equation as a source term. Treating the fluctuations to linear order, corresponding to an RPA-like approximation, one has

$$\begin{aligned}&\frac{\partial}{\partial t}\Delta n_{\mathbf{p}} + (\mathbf{v}_{\mathbf{p}}\nabla_{\mathbf{r}})\Delta\tilde{n}_{\mathbf{p}} \\ &\quad + e(\mathbf{E}+\mathbf{E}^{\rm CS})\nabla_{\mathbf{p}}\Delta n_{\mathbf{p}} + e(\mathbf{v}_{\mathbf{p}}\times\boldsymbol{\Delta}\mathbf{B}^{\rm CS})\nabla_{\mathbf{p}}\delta\tilde{n}_{\mathbf{p}} \\ &\quad\quad + e(\Delta\mathbf{E}+\Delta\mathbf{E}^{\rm CS})\nabla_{\mathbf{p}} n_{\mathbf{p}} - S'_{\mathbf{p}}\{\Delta n_{\mathbf{p}}\} = \Delta J_{\mathbf{p}}.\end{aligned} \tag{8}$$

The left-hand side of this equation describes the evolution of $\Delta n_{\mathbf{p}}$ due to the CF kinematics and scattering. The latter is described by the linearized collision integral $S'_{\mathbf{p}}\{\Delta n_{\mathbf{p}}\}$. In the relaxation-time approximation, one has

$$S'_{\mathbf{p}}\{\Delta n_{\mathbf{p}}\} = \frac{1}{\tau_{\rm tr}}\left\{\Delta\tilde{n}_{\mathbf{p}} - \int\frac{d\theta_{\mathbf{p}}}{2\pi}\Delta\tilde{n}_{\mathbf{p}}\right\}. \tag{9}$$

The fluctuations are driven by the source term $\Delta J_{\mathbf{p}}$, characterized by a zero average and correlator[17]

$$\begin{aligned}&\langle\Delta J_{\mathbf{p}}(\mathbf{r},t)\Delta J_{\mathbf{p}'}(\mathbf{r}',t')\rangle = \Omega\,\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t') \\ &\quad\times\{\delta_{\mathbf{p}\mathbf{p}'}\sum_{\mathbf{p}_1} W_{\mathbf{p}\mathbf{p}_1}[n_{\mathbf{p}_1}(1-n_{\mathbf{p}}) + n_{\mathbf{p}}(1-n_{\mathbf{p}_1})] \\ &\quad\quad - W_{\mathbf{p}\mathbf{p}'}[n_{\mathbf{p}'}(1-n_{\mathbf{p}}) + n_{\mathbf{p}}(1-n_{\mathbf{p}'})]\}.\end{aligned} \tag{10}$$

Both for the linearized collision integral and for the source term, only the contribution of impurity scattering was kept, reflecting the assumption that $\ell_{\rm tr} \ll L_{\rm cf-cf}, L_{\rm cf-ph}$. The source term turns out to enter into subsequent calculations only in the combination

$$\boldsymbol{\Delta}\mathbf{J} = \frac{e\tau_{\rm tr}}{\Omega}\sum_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}\Delta J_{\mathbf{p}} \tag{11}$$

with zero average and variance

$$\begin{aligned}&\langle\Delta J^{\alpha}(\mathbf{r},t)\Delta J^{\beta}(\mathbf{r}',t')\rangle = 2\sigma_{\rm CF} \\ &\quad\times\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t')\delta^{\alpha\beta}\int d\epsilon\, n_{\epsilon}(1-n_{\epsilon}).\end{aligned} \tag{12}$$

Here, n_ϵ denotes the part of the average distribution function $n_{\mathbf{p}}$ which is isotropic in momentum space. The contribution due to the non-isotropic part is negligibly small in the diffusive regime.

Density fluctuations

$$\Delta\rho = \frac{e}{\Omega}\sum_{\mathbf{p}}\Delta n_{\mathbf{p}} \tag{13}$$

cause fluctuations in the Chern-Simons magnetic field,

$$\boldsymbol{\Delta}\mathbf{B}^{\rm CS} = \frac{2h}{e^2}\,\Delta\rho\,\hat{\mathbf{z}}. \tag{14}$$

Unlike the average Chern-Simons magnetic field at half filling, these fluctuations are no longer canceled by the applied magnetic field and must therefore be included in the Boltzmann-Langevin equation. Both $\Delta\mathbf{B}^{\rm CS}$ and the fluctuations in the physical and Chern-Simons electric fields,

$$\Delta\phi^{\rm ind}(\mathbf{r}) = \int d\mathbf{r}'\frac{\Delta\rho(\mathbf{r}')}{\epsilon|\mathbf{r}-\mathbf{r}'|}, \tag{15}$$

$$\boldsymbol{\Delta}\mathbf{E}^{\rm CS} = \frac{2h}{e^2}(\hat{\mathbf{z}}\times\boldsymbol{\Delta}\mathbf{j}), \tag{16}$$

with $\Delta\mathbf{E} = -\nabla_{\mathbf{r}}(\Delta\phi^{\rm ext} + \Delta\phi^{\rm ind})$ need to be determined self consistently. Here,

$$\boldsymbol{\Delta}\mathbf{j} = \frac{e}{\Omega}\sum_{\mathbf{p}}\mathbf{v}_{\mathbf{p}}\Delta\tilde{n}_{\mathbf{p}} \tag{17}$$

denotes the fluctuations in the current density.

We briefly comment on the range of validity of the Boltzmann-Langevin approach to current noise. The use of semiclassical transport theory restricts us to frequencies $\hbar\omega \ll E_F$ and wavevectors $q \ll k_F$. The diffusive regime considered here requires the more stringent conditions $\omega \ll 1/\tau_{\rm tr}$ and $q \ll 1/\ell_{\rm tr}$. Moreover, the sample should be large compared to the phase-coherence length L_ϕ. However, for electrons it turned out that phase coherence does not affect the shot-noise power as long as $\omega \ll eV/\hbar$. It is natural to expect the same for the present problem.

In the diffusive limit, the Boltzmann-Langevin equation can be reduced to hydrodynamic equations for the the macroscopic quantities $\Delta\rho$ and $\boldsymbol{\Delta}\mathbf{j}$. Here we only sketch the derivation. Details can be found in appendix A. Assuming that the fluctuations of the distribution function occur on scales large compared to the elastic mean-free path, $\Delta n_{\mathbf{p}}$ is mostly isotropic with respect to the directions of the momentum $\mathbf{p}$ with a small anisotropic part. Accordingly, we decompose

$$\Delta n_{\mathbf{p}} = \Delta n_\epsilon + \mathbf{v_p}\mathbf{\Delta f}_\epsilon \quad (18)$$
$$\Delta J_{\mathbf{p}} = \Delta J_\epsilon + \mathbf{v_p}\mathbf{\Delta J}_\epsilon. \quad (19)$$

These decompositions are inserted into (8) and the Boltzmann-Langevin equation is split into its isotropic and anisotropic parts. Upon multiplying the isotropic part by e/Ω and summing over all momenta $\mathbf{p}$, one finds that density and current fluctuations must satisfy the continuity equation

$$\frac{\partial}{\partial t}\Delta\rho + \nabla_{\mathbf{r}}\mathbf{\Delta j} = 0. \quad (20)$$

Upon multiplying the anisotropic part of the Boltzmann-Langevin equation by $(e\tau_{\rm tr}/\Omega)\mathbf{v_p}$ and summing over all $\mathbf{p}$, one obtains the response equation

$$\begin{aligned}\mathbf{\Delta j} = \mathbf{\Delta J} - D^*\nabla_{\mathbf{r}}\Delta\rho - \Delta D^*\nabla_{\mathbf{r}}\rho - \frac{e\tau_{\rm tr}}{m^*}(\mathbf{\Delta B}^{\rm CS}\times\mathbf{j})\\ +\sigma_{\rm CF}(\mathbf{\Delta E}+\mathbf{\Delta E}^{\rm CS}) + \Delta\sigma_{\rm CF}(\mathbf{E}+\mathbf{E}^{\rm CS}). \quad (21)\end{aligned}$$

Here, $D^* = D(1+F_0)$ denotes a renormalized diffusion constant with $D = v_F\ell_{tr}/2$ and F_0 the Landau parameter. The various terms in this equation have obvious interpretations. The first term on the right-hand side describes the current fluctuations due to the statistical nature of the impurity scattering. The next two terms represent fluctuations in the diffusion current associated with fluctuations in the density and the diffusion constant. These contributions are renormalized by the quasiparticle interaction. The fourth term reflect fluctuations in the Lorentz force due to fluctuations in the Chern-Simons magnetic field. This term was previously discussed in Ref. 21. The last two terms describe fluctuations in the response to the electric fields due to fluctuations in the electric fields and the conductivity, respectively.

How are the current fluctuations affected by the Chern-Simons fields? To answer this question, it is instructive to derive the analogous response equation for classical electrons in a magnetic field. This calculation is also done in appendix A and one finds

$$\begin{aligned}\mathbf{\Delta j} = \mathbf{\Delta J} - D^*\nabla_{\mathbf{r}}\Delta\rho - \Delta D^*\nabla_{\mathbf{r}}\rho\\ +\sigma\mathbf{\Delta E} + \Delta\sigma\mathbf{E} - \frac{e\tau_{\rm tr}}{m^*}(\mathbf{B}\times\mathbf{\Delta j}). \quad (22)\end{aligned}$$

One observes that the response equations for composite fermions and electrons differ in two points:

- The response equation for composite fermions includes additional terms involving the Chern-Simons fields and their fluctuations.
- The response equation for composite fermions lacks a Lorentz-force term due to the applied magnetic field.

We will now show that the two response equations are equivalent to leading order despite these seeming differences.

First, we rewrite the term in (21) involving $\mathbf{\Delta E}^{\rm CS}$ using Eq. (16) and find

$$\sigma_{\rm CF}\mathbf{\Delta E}^{\rm CS} = -\frac{e\tau_{\rm tr}}{m^*}(\mathbf{B}_{1/2}\times\mathbf{\Delta j}), \quad (23)$$

where it was used that at half filling $eB_{1/2}\tau_{\rm tr}/m^* = -(2h/e^2)\sigma_{\rm CF}$. Hence, this term in the response equation for composite fermions reproduces the Lorentz-force term for electrons. This is analogous to the fact that the Chern-Simons electric field leads to the finite Hall resistivity in the derivation of the physical resistivity tensor from the Boltzmann equation (cf., sec. II A).

We are now left with two additional terms in the response equation for composite fermions without analog in the case of electrons in a magnetic field. We first discuss the term involving the fluctuations in the conductivity. These fluctuations arise due to fluctuations in the density of the sample. For a quadratic dispersion, the effective mass is independent of the density so that

$$\Delta\sigma_{\rm CF} = \frac{e\Delta\rho\,\tau_{\rm tr}}{m^*}. \quad (24)$$

For arbitrary dispersions, there is an additional contribution arising from fluctuations of the effective mass with density. Generally, the terms due to fluctuations in the diffusion constant and the conductivity are neglected in the response equation (21) because they are of order $\max\{T, eV\}/\mu$ relative to the leading contributions. Here, it is instructive to keep them because this allows one to show that the term involving $\mathbf{\Delta B}^{\rm CS}$ is of the same order and hence can be neglected. In fact, replacing the Chern-Simons fields and $\Delta\sigma_{\rm CF}$ by their explicit expressions, one finds that the two terms actually cancel exactly for a quadratic dispersion,

$$\Delta\sigma_{\rm CF}\mathbf{E}^{\rm CS} - \frac{e\tau_{\rm tr}}{m^*}(\mathbf{\Delta B}^{\rm CS}\times j) = 0. \quad (25)$$

This shows that the term involving $\mathbf{\Delta B}^{\rm CS}$ is also of order $\max\{T, eV\}/\mu$ relative to the leading terms and can be neglected. Of course, the two terms do not cancel exactly in the general case of a non-quadratic dispersion.

The contribution of $\mathbf{\Delta B}^{\rm CS}$ to fluctuations in the Hall voltage has been discussed in detail in Ref. 21. There it was argued that these Hall-voltage fluctuations could be a signature of the presence of Chern-Simons fields in the sample. The present approach shows that the dominant contribution to Hall voltage fluctuations does not come from $\mathbf{\Delta B}^{\rm CS}$, but instead from the fluctuating Chern-Simons electric field. The latter mechanism cannot be used to verify the presence of fluctuating gauge fields in the sample because it simply reflects the presence of a finite Hall resistivity, due to which any current fluctuation causes fluctuations in the Hall voltage.

We briefly consider magnetic fields away from $\nu = 1/2$. In this case, the semiclassical approach to transport is valid as long as Shubnikov-de Haas oscillations of the composite fermions are negligible. Away from

$\nu = 1/2$, the CF's experience an effective magnetic field $B^* = B - B_{1/2}$ which leads to the additional term $-(e\tau_{\rm tr}/m^*)(\mathbf{B}^* \times \mathbf{\Delta j})$ in the response equation. When combined with the term due to fluctuations in the Chern-Simons electric field, one obtains a Lorentz-force term appropriate for the full externally applied magnetic field B.

As a result, we conclude that the density and current fluctuations of composite fermion in the diffusive regime do not differ to leading order from the fluctuations of classical electrons in the external magnetic field. Neglecting the terms involving ΔD and $\Delta\sigma$, we find for composite fermions near $\nu = 1/2$

$$\mathbf{\Delta j} = \mathbf{\Delta J} - D^* \nabla_{\mathbf{r}} \Delta\rho + \sigma_{\rm CF} \mathbf{\Delta E} - \frac{e\tau_{\rm tr}}{m^*}(\mathbf{B} \times \mathbf{\Delta j}). \quad (26)$$

Of course, the analogy between the response equations for classical electrons and composite fermions concerns the form of the response equation. The values of the phenomenological constants such as m^* and $\tau_{\rm tr}$ entering into the response equation do not coincide.

III. LOW-FREQUENCY NOISE POWER

In this section, the general framework derived above is employed to compute the equilibrium and excess noise power near $\nu = 1/2$ in the Corbino disc. We will first consider the case of zero frequency. The question of finite frequency is discussed at the end of this section. The shot-noise power is computed for the two regimes $\ell_{\rm tr} \ll L \ll L_{\rm cf-cf} \ll L_{\rm cf-ph}$ and $\ell_{\rm tr} \ll L_{\rm cf-cf} \ll L \ll L_{\rm cf-ph}$. For samples larger than $L_{\rm cf-ph}$, the shot-noise power vanishes and there is only equilibrium noise.

For the Corbino disc we use a coordinate system such that the x direction points in the radial direction and the y direction in the angular direction around the disc. Then, at zero frequency, the continuity equation implies that

$$\Delta I_x = \int_{-L_y/2}^{L_y/2} dy\, \Delta j_x = \frac{1}{L_x} \int_\Omega dx\, dy\, \Delta j_x \quad (27)$$

$$\Delta I_y = \int_{-L_x/2}^{L_x/2} dx\, \Delta j_y = \frac{1}{L_y} \int_\Omega dx\, dy\, \Delta j_y. \quad (28)$$

Using the response equation (26) yields

$$\Delta I_x = \frac{1}{L_x} \int_\Omega dx\, dy\, \Delta J_x + \frac{2h}{e^2} \sigma_{\rm CF} \frac{L_y}{L_x} \Delta I_y - \frac{1}{L_x} \int_{-L_y/2}^{L_y/2} dy\, \left[D^* \Delta\rho + \sigma_{\rm CF} \Delta\phi\right]_{x=-L_x/2}^{x=L_x/2} \quad (29)$$

and

$$\Delta I_y = \frac{1}{L_y} \int_\Omega dx\, dy\, \Delta J_y - \frac{2h}{e^2} \sigma_{\rm CF} \frac{L_x}{L_y} \Delta I_x - \frac{1}{L_y} \int_{-L_x/2}^{L_x/2} dx\, \left[D^* \Delta\rho + \sigma_{\rm CF} \Delta\phi\right]_{y=-L_y/2}^{y=L_y/2}. \quad (30)$$

Here, we introduced the notation $[f(x)]_{x=a}^{x=b} = f(b) - f(a)$. We remark that the Hall current around the loop can in principle be measured by means of the associated magnetic moment. The quantities in the square brackets in Eqs. (29) and (30) are proportional to the electrochemical potential differences across the sample in the x and y direction, respectively, if the distribution functions on the respective edges of the sample are the equilibrium distribution function. This follows from the standard result of Fermi-liquid theory[19]

$$\Delta\rho = \frac{\Delta\rho}{\Delta\mu} \Delta\mu = \frac{eN(0)}{1 + F_0} \Delta\mu \quad (31)$$

and the Einstein relation.

In the Corbino-disc geometry, the electrochemical potential difference in the y direction (i.e., around the disc) must vanish due to periodicity. Hence, the last term in Eq. (30) is zero. Moreover, since we are interested in the intrinsic current noise originating in the sample, we assume that the voltage source maintains a fixed electrochemical potential difference across the sample. For the Corbino-disc geometry, this implies that also the last term in Eq. (29) vanishes.

It is now simple to solve the equations (29) and (30) for the current fluctuations,

$$\Delta I_x = \frac{1}{1 + (2h\sigma_{\rm CF}/e^2)^2} \times \frac{1}{L_x} \int_\Omega dx\, dy\, \left\{\Delta J_x + (2h\sigma_{\rm CF}/e^2)^2 \Delta J_y\right\} \quad (32)$$

$$\Delta I_y = \frac{1}{1 + (2h\sigma_{\rm CF}/e^2)^2} \times \frac{1}{L_y} \int_\Omega dx\, dy\, \left\{-(2h\sigma_{\rm CF}/e^2)^2 \Delta J_x + \Delta J_y\right\}. \quad (33)$$

The associated zero-frequency noise powers are defined as

$$S_\alpha = 2 \int dt\, \langle \Delta I_\alpha(t) \Delta I_\alpha(t') \rangle \quad (34)$$

with $\alpha = x, y$. Using Eq. (12), one finds

$$S_x = 4 \frac{\sigma_{\rm CF} L_y/L_x}{1 + (2h\sigma_{\rm CF}/e^2)^2} \int_\Omega \frac{d\mathbf{r}}{\Omega} \int d\epsilon\, n_\epsilon (1 - n_\epsilon) \quad (35)$$

$$S_y = 4 \frac{\sigma_{\rm CF} L_x/L_y}{1 + (2h\sigma_{\rm CF}/e^2)^2} \int_\Omega \frac{d\mathbf{r}}{\Omega} \int d\epsilon\, n_\epsilon (1 - n_\epsilon). \quad (36)$$

Note that the noise power S_x of the longitudinal current between edges differs from the noise power S_y of the Hall current around the loop only by geometrical factors. The prefactors are simply the conductances of the Corbino disc.

If the composite-fermion distribution function is in (local) equilibrium, we can further simplify the expression for the noise power to

$$S_x = 4\frac{\sigma_{\rm CF} L_y/L_x}{1+(2h\sigma_{\rm CF}/e^2)^2}\int_\Omega \frac{d\mathbf{r}}{\Omega}\, T_{\rm cf}(\mathbf{r}) \tag{37}$$

$$S_y = 4\frac{\sigma_{\rm CF} L_x/L_y}{1+(2h\sigma_{\rm CF}/e^2)^2}\int_\Omega \frac{d\mathbf{r}}{\Omega}\, T_{\rm cf}(\mathbf{r}) \tag{38}$$

in terms of the local composite-fermion temperature $T_{\rm cf}(\mathbf{r})$. In particular, one immediately recovers from this expression the standard result for equilibrium noise. Another simple application are samples with $L \gg L_{\rm cf-ph}$. In this case, the composite-fermion temperature is everywhere equal to the phonon temperature. Hence, there is only equilibrium noise and shot noise vanishes. Below, we will derive the shot-noise power for the nontrivial regimes with $L \ll L_{\rm cf-ph}$.

Eq. (36) expresses the zero-frequency noise power in terms of the isotropic part of the average distribution function n_ϵ. To linear order in the applied bias and in the quasiparticle interaction, this quantity satisfies a modified diffusion equation,

$$D\nabla^2_{\mathbf{r}} n_{\epsilon-\delta(x,y)} + S^{\rm cf-cf}_{\mathbf{p}}\{n_{\mathbf{p}}\} = 0. \tag{39}$$

where $\delta(x,y) = \sum_{\mathbf{p}'} f_{\mathbf{p}\mathbf{p}'}\delta n_{\mathbf{p}'} + e\phi$. The quantity δ depends only on the density distribution in the sample and is independent of ϵ. The derivation of this equation from the Boltzmann equation is sketched in appendix B. The solution of (39) simplifies for the Corbino-disc geometry when using the fact that $\sigma_{xy}/\sigma_{xx} \gg 1$. In this limit, the inner and outer edges are, to a good approximation, equipotential lines, even if the contacts to the leads are local. This implies that the distribution function becomes essentially independent of the angular direction y. With this observation, we can state the appropriate boundary conditions for Eq. (39). The battery provides an electrochemical-potential difference eV between the inner and the outer edge,

$$(\mu + e\phi)_{x=L_x/2} - (\mu+e\phi)_{x=-L_x/2} = eV. \tag{40}$$

Hence, the chemical potentials μ_i and μ_o at the inner and outer edges are

$$\mu_o = \mu - e\phi_o + \frac{eV}{2} \tag{41}$$

$$\mu_i = \mu - e\phi_i - \frac{eV}{2}, \tag{42}$$

with μ a constant. Since the distribution function in the leads is in equilibrium, we have

$$n_{\mathbf{p}}(x=\pm L_x/2) = f_{\mu\pm eV/2}(\tilde\epsilon_{\mathbf{p}} + e\phi(\pm L_x/2)), \tag{43}$$

where $f_\mu(E)$ denotes the Fermi-Dirac distribution. From this, we obtain the boundary condition

$$n_{\epsilon-\delta(x=\pm L_x/2)}(x=\pm L_x/2) = f_{\mu\pm eV/2}(\epsilon) \tag{44}$$

for the diffusion equation (39).

We first specify to the limit of weak CF-CF scattering, $L \ll L_{\rm cf-cf}$, where we can neglect the collision integral $S^{\rm cf-cf}_{\mathbf{p}}$ in the diffusion equation (39). In this limit, the distribution function becomes

$$\begin{aligned} n_\epsilon = &\left[f_{\mu+eV/2}(\epsilon+\delta(x)) - f_{\mu-eV/2}(\epsilon+\delta(x))\right]\frac{x}{L} \\ &+\frac{1}{2}\left[f_{\mu+eV/2}(\epsilon+\delta(x)) + f_{\mu-eV/2}(\epsilon+\delta(x))\right]. \end{aligned} \tag{45}$$

Note that both the induced fluctuations in the physical electric field and the effects of nontrivial Fermi-liquid parameters are contained entirely in the quantity $\delta(x)$. Inserting this solution for the average distribution function into Eq. (36), one obtains the final result

$$S_\alpha = 4G_\alpha\left\{\frac{2}{3}T + \frac{eV}{6}\coth\frac{eV}{2T}\right\}. \tag{46}$$

Here we defined the conductances

$$\begin{aligned} G_x &= \frac{\sigma_{\rm CF} L_y/L_x}{1+(2h\sigma_{\rm CF}/e^2)^2} \\ G_y &= \frac{\sigma_{\rm CF} L_x/L_y}{1+(2h\sigma_{\rm CF}/e^2)^2} \end{aligned} \tag{47}$$

of the Corbino disc. Interestingly, the induced electric field fluctuations and nontrivial Fermi-liquid parameters have no effect on the noise power and we obtain precisely the same result for S_x as for usual diffusive contacts in the absence of a magnetic field. In the present problem, there are additional fluctuations S_y in the Hall current around the loop due to the finite Hall conductivity of the sample. The magnitudes of the fluctuations in the longitudinal and Hall currents differ only by geometric factors.

We now turn to the regime of strong CF-CF scattering, $L \gg L_{\rm cf-cf}$. In this regime, CF-CF scattering locally equilibrates the distribution function so that n_ϵ can be parameterized by a local chemical potential $\mu(\mathbf{r})$ and a local temperature $T_{\rm cf}(\mathbf{r})$, which in general does not coincide with the phonon temperature because of Joule heating. An equation for $T_{\rm cf}(\mathbf{r})$ can be derived from the diffusion equation (39). Here we follow an alternative, more direct approach.[10] The heat current carried by the composite fermions due to a temperature gradient is

$$\mathbf{j}^q = -\hat\sigma\frac{\pi^2 k_B^2}{6e^2}\nabla_{\mathbf{r}} T^2_{\rm cf}, \tag{48}$$

where $\hat\sigma$ denotes the conductivity tensor. In principle, there is also a thermoelectric contribution to the heat current, which, however, can be neglected. Joule heating acts as a source for the heat current so that

$$\nabla_{\mathbf{r}}\mathbf{j}^q = -\mathbf{j}^e\nabla_{\mathbf{r}}(\phi + \frac{1}{e}\mu), \tag{49}$$

where $\mathbf{j}^e$ denotes the charge current. These equations are valid for arbitrary Fermi-liquid parameters. For the Corbino disc, one readily finds that the Joule heating is

$$-\mathbf{j}^e\nabla_{\mathbf{r}}(\phi+\frac{1}{e}\mu) = \sigma_{xx}\left(\frac{V}{L_x}\right)^2. \tag{50}$$

Hence, by inserting Eq. (48) into (49), we obtain an equation for the composite-fermion temperature,

$$\nabla_{\mathbf{r}}{}^2 T_{\rm cf}^2 + \frac{6e^2}{\pi^2 k_B^2}\left(\frac{V}{L_x}\right)^2 = 0. \tag{51}$$

We assume that the electrons are in good thermal contact with the phonon bath in the leads. Hence, we need to solve Eq. (51) subject to the boundary condition $T_{\rm cf}(x = \pm L_x/2) = T$. Specifying for simplicity to $eV \gg T$, we obtain for the CF temperature profile

$$T_{\rm cf}(x) \simeq \frac{\sqrt{3}}{2\pi} eV \sqrt{1 - \left(\frac{2x}{L_x}\right)^2}. \tag{52}$$

Hence, the noise power in the regime $L_{\rm cf-cf} \ll L \ll L_{\rm cf-ph}$ becomes

$$S_\alpha = \frac{\sqrt{3}}{2} e G_\alpha V. \tag{53}$$

The result for S_x is again the same as for electrons in zero-magnetic field, when written in terms of the average current flowing through the device. The fluctuations in the Hall current differs only by geometric factors from the fluctuations of the longitudinal current.

Finally, we address the question of finite frequency. Our treatment remains valid at finite frequency as long as the time derivative of the density fluctuations can be neglected in the diffusion equation. For three-dimensional samples, this is valid for frequencies smaller than the Maxwell frequency $4\pi\sigma_{xx}$. In two-dimensional systems, this frequency is smaller because screening is less effective. We estimate this frequency by considering

$$\mathbf{k}\Delta\mathbf{j} = \mathbf{k}\hat{\sigma}\Delta\mathbf{E} = -i2\pi\sigma_{xx}k(1/\epsilon)\Delta\rho, \tag{54}$$

where we used Maxwell's equation. Hence, the zero-frequency result remains good for frequencies

$$\omega \ll 2\pi\sigma_{xx}k/\epsilon \approx 10^8\text{Hz}. \tag{55}$$

The typical scale for the wavevector is set by the system size for which we assumed 10^{-4}m for the numerical estimate. We note that the condition $\omega < 1/\tau_{\rm tr}$ needed for the validity of the diffusive approximation is typically weaker than this requirement.

IV. THERMAL CORRELATORS AT FINITE WAVEVECTOR AND FREQUENCY

The Boltzmann-Langevin formalism can also be applied to compute thermal density-density and current-current correlators at finite frequency and wavevector in the diffusive regime. These correlators describe the actual density and current fluctuations in the sample. Once the correlators for the currents and densities are known, the correlators of the Chern-Simons electric and magnetic fields and for the physical electric field can be obtained from Eqs. (14), (15), and (16).

Writing the response equation (26) in frequency and momentum space, we have

$$\mathbf{\Delta j} = \mathbf{\Delta J} - iD^*\mathbf{k}\Delta\rho + \sigma_{\rm CF}\mathbf{\Delta E} - \frac{e\tau_{\rm tr}}{m^*}(\mathbf{B}\times\mathbf{\Delta j}). \tag{56}$$

The density and electric-field fluctuations can be eliminated from this equation using the continuity equation

$$\omega\Delta\rho = \mathbf{k}\mathbf{\Delta j} \tag{57}$$

and Maxwell's equation

$$\epsilon\mathbf{\Delta E} = -i2\pi\hat{\mathbf{k}}\Delta\rho, \tag{58}$$

where $\hat{\mathbf{k}}$ is the unit vector in the direction of $\mathbf{k}$. Decomposing the current into its components parallel and perpendicular to the wavevector $\Delta j_\parallel = \hat{\mathbf{k}}\mathbf{\Delta j}$ and $\Delta j_\perp = \hat{\mathbf{k}}(\hat{\mathbf{z}}\times\mathbf{\Delta j})$, one finds

$$\Delta j_\parallel = \Delta J_\parallel - i\frac{D^*k^2}{\omega}\Delta j_\parallel - i\frac{2\pi\sigma_{\rm CF}|k|}{\epsilon\omega}\Delta j_\parallel + \omega_c\tau_{\rm tr}\Delta j_\perp \tag{59}$$

and

$$\Delta j_\perp = \Delta J_\perp - \omega_c\tau_{\rm tr}\Delta j_\parallel. \tag{60}$$

These equations are readily solved and one obtains

$$\Delta j_\parallel = \frac{\Delta J_\parallel + \omega_c\tau_{\rm tr}\Delta J_\perp}{1 + (\omega_c\tau_{\rm tr})^2 + (i/\omega)[D^*k^2 + 2\pi\sigma_{\rm CF}|k|/\epsilon]} \tag{61}$$

and

$$\Delta j_\perp = \Delta J_\perp - \frac{\omega_c\tau_{\rm tr}(\Delta J_\parallel + \omega_c\tau_{\rm tr}\Delta J_\perp)}{1 + (\omega_c\tau_{\rm tr})^2 + (i/\omega)[D^*k^2 + 2\pi\sigma_{\rm CF}|k|/\epsilon]}. \tag{62}$$

From the definition of the source current $\mathbf{\Delta J}$, one obtains in thermal equilibrium

$$\begin{aligned}\langle\Delta J_\parallel\Delta J_\parallel\rangle_{\omega,\mathbf{k}} &= \langle\Delta J_\perp\Delta J_\perp\rangle_{\omega,\mathbf{k}} = 2T\sigma_{\rm CF}\\ \langle\Delta J_\parallel\Delta J_\perp\rangle_{\omega,\mathbf{k}} &= 0.\end{aligned} \tag{63}$$

Here we use the notation $\langle fg\rangle_{\omega,\mathbf{k}} = \langle f(\omega,\mathbf{k})g(-\omega,-\mathbf{k})\rangle$. This yields the current-current correlators

$$\langle\Delta j_\parallel\Delta j_\parallel\rangle_{\omega,\mathbf{k}} = \frac{2T\sigma_{xx}\omega^2}{\omega^2 + [D_{xx}^*k^2 + 2\pi\sigma_{xx}k/\epsilon]^2}, \tag{64}$$

and

$$\langle\Delta j_\perp\Delta j_\perp\rangle_{\omega,\mathbf{k}} = 2T\sigma_{xx} \times\left\{1 + \frac{[D_{xy}^*k^2 + 2\pi\sigma_{xy}k]^2}{\omega^2 + [D_{xx}^*k^2 + 2\pi\sigma_{xx}k/\epsilon]^2}\right\}. \tag{65}$$

Here, $\sigma_{\alpha\beta}$ denote the components of the physical conductivity tensor and we defined a diffusion tensor by $\sigma_{\alpha\beta} = e^2 N(0) D_{\alpha\beta}$. Note that these correlators also corroborate the estimate for the validity of the zero-frequency approximation at the end of the previous section. Using the continuity equation, one finds for the density-density correlator

$$\langle \Delta\rho\Delta\rho \rangle_{\omega,\mathbf{k}} = \frac{2T\sigma_{xx}k^2}{\omega^2 + [D^*_{xx}k^2 + 2\pi\sigma_{xx}k/\epsilon]^2}. \tag{66}$$

All of these correlators for composite fermions are identical to those for semiclassical electrons in the external magnetic field. From the Boltzmann-Langevin approach, this result follows directly for the thermal correlators. The analogous result for the retarded and advanced correlators follows from the fluctuation-dissipation theorem combined with the Kramers-Kronig relations.

V. CONCLUSIONS AND SUMMARY

In this paper, we have studied non-equilibrium density and current fluctuations in the half-filled Landau level, employing the composite-fermion picture and focusing on the diffusive regime. By deriving a Boltzmann-Langevin equation for composite fermions, including both the coupling to the Chern-Simons fields and arbitrary quasiparticle interactions, we found that, *to leading order, the density and current fluctuations for diffusive samples near* $\nu = 1/2$ *are equivalent to those of semiclassical electrons in the externally applied magnetic field.* Current fluctuations associated with fluctuations in the Chern-Simons magnetic field are suppressed by $\max\{eV, T\}/\mu$ relative to the leading contributions. Fluctuations in the Chern-Simons electric field are important and reproduce the term arising due to the large Lorentz force for semiclassical electrons. One consequence of this is that fluctuations in the Hall voltage are dominated by fluctuations in the Chern-Simons electric field, reflecting the large Hall conductivity of the sample, rather than fluctuations in the Chern-Simons magnetic field. This is in contrast to previous suggestions.[21]

The general results for density and current fluctuations can be used to compute the shot-noise power in the half-filled Landau level. For the Corbino-disc geometry, we found that the shot-noise power at $\nu = 1/2$, when expressed in terms of the average current flowing through the device, equals that for metallic systems in low magnetic fields. This implies that diffusive shot noise near $\nu = 1/2$ remains unaffected by quasiparticle interactions and by the coupling to the Chern-Simons fields. We note in passing that the insensitivity of shot noise to Fermi-liquid corrections also holds for regular metallic contacts. Shot noise in a two-terminal conductor near $\nu = 1/2$ would also be interesting for comparison with results for fractional-quantum-Hall states,[22] but the corresponding calculations are complicated by the fact that, in this geometry, most of the resistance is associated with the contacts between sample and leads. Finally, we find that shot noise in the half-filled Landau level depends sensitively on the ratio of the sample size to the CF-CF and the CF-phonon scattering lengths. Our results suggest that shot noise can be used to measure these important length scales.

ACKNOWLEDGMENTS

I would like to acknowledge support by a scholarship of the Minerva Foundation, Munich, a Minerva project grant, and grant no. 95-250/1 of the U.S.-Israel Binational Science Foundation. I enjoyed very useful and informative discussions with Rafi de-Picciotto, Misha Reznikov, and Ady Stern.

APPENDIX A: NOISE IN DIFFUSIVE SYSTEMS IN PRESENCE OF MAGNETIC FIELD

In this appendix we provide details of the calculations sketched in Sec. II B. In particular, we will show how to derive the continuity and response equations from the Boltzmann-Langevin equation including the terms arising from fluctuations in the diffusion constant which are usually neglected but turned out to be conceptually important in the present context. The present calculation also includes the effects of a non-vanishing quasiparticle interaction. We will restrict our calculation to strictly two-dimensional samples and consider a system of classical electrons in the diffusive regime. The derivation of the response equation (21) for composite fermions at $\nu = 1/2$ is completely analogous.

The Boltzmann-Langevin equation is

$$\frac{\partial}{\partial t}\Delta n_{\mathbf{p}} + \mathbf{v_p}\nabla_{\mathbf{r}}\Delta\tilde{n}_{\mathbf{p}} + e\mathbf{E}\nabla_{\mathbf{p}}\Delta n_{\mathbf{p}} + e(\mathbf{v_p}\times\mathbf{B})\nabla_{\mathbf{p}}\Delta\tilde{n}_{\mathbf{p}} + e\Delta\mathbf{E}\nabla_{\mathbf{p}}n_{\mathbf{p}} - S'_{\mathbf{p}}\{\Delta n_{\mathbf{p}}\} = \Delta J_{\mathbf{p}}. \tag{A1}$$

We start by introducing the decompositions (18) and (19) into the Boltzmann-Langevin equation. Collecting the resulting terms in the Boltzmann-Langevin equation which are isotropic in $\mathbf{p}$, we find

$$\frac{\partial}{\partial t}\Delta n_{\epsilon} + \mathbf{v_p}\nabla_{\mathbf{r}}(\mathbf{v_p}\boldsymbol{\Delta}\tilde{\mathbf{f}}_{\epsilon}) + e\mathbf{E}\nabla_{\mathbf{p}}(\mathbf{v_p}\boldsymbol{\Delta}\mathbf{f}_{\epsilon}) + e(\mathbf{v_p}\times\mathbf{B})\nabla_{\mathbf{p}}\Delta\tilde{n}_{\epsilon} + e\Delta\mathbf{E}\nabla_{\mathbf{p}}(\mathbf{v_p}\mathbf{f}_{\epsilon}) = 0. \tag{A2}$$

Here, we used that ΔJ_{ϵ} has zero average and variance. To derive an equation in terms of the charge and current densities (13) and (17), we multiply this equation by e/Ω and sum over all $\mathbf{p}$. We will now show that the equation will then reduce to the continuity equation (20). One immediately finds that the first two terms in (A2) give the

time derivative of the density and the divergence of the current density. The electric-field term vanishes because

$$\begin{aligned}
&\frac{e^2}{\Omega}\sum_{\mathbf{p}}\mathbf{E}\nabla_{\mathbf{p}}(\mathbf{v_p}\boldsymbol{\Delta}\mathbf{f}_\epsilon) \\
&\quad = \frac{e^2}{\Omega}\sum_{\mathbf{p}}\mathbf{E}\left\{\frac{\boldsymbol{\Delta}\mathbf{f}_\epsilon}{m^*}+\mathbf{v_p}\left(\mathbf{v_p}\frac{\partial\boldsymbol{\Delta}\mathbf{f}_\epsilon}{\partial\epsilon}\right)\right\} \\
&\quad = \frac{e^2}{m^*}N(0)\int d\epsilon\,\mathbf{E}\left\{\boldsymbol{\Delta}\mathbf{f}_\epsilon+\epsilon\frac{\partial\boldsymbol{\Delta}\mathbf{f}_\epsilon}{\partial\epsilon}\right\} \\
&\quad = 0
\end{aligned}\tag{A3}$$

The term involving the fluctuations of the electric field vanishes for the same reason. Finally, the magnetic-field term obviously vanishes because of the vector structure. This completes the derivation of the continuity equation from the Boltzmann-Langevin equation.

Next, we collect the terms in the Boltzmann-Langevin equation which are anisotropic in the momentum $\mathbf{p}$,

$$\begin{aligned}
&\mathbf{v_p}\nabla_{\mathbf{r}}\Delta\tilde{n}_\epsilon + e\mathbf{E}\nabla_{\mathbf{p}}\Delta n_\epsilon + e(\mathbf{v_p}\times\mathbf{B})\nabla_{\mathbf{p}}(\mathbf{v_p}\boldsymbol{\Delta}\tilde{\mathbf{f}}_\epsilon) \\
&\qquad + e\Delta\mathbf{E}\nabla_{\mathbf{p}}n_\epsilon + \frac{1}{\tau_{\rm tr}}(\mathbf{v_p}\boldsymbol{\Delta}\tilde{\mathbf{f}}_\epsilon) = \Delta J_{\mathbf{p}}.
\end{aligned}\tag{A4}$$

Here, we neglected the time derivative relative to the collision integral. To derive the response equation (22), we multiply by $(e\tau_{\rm tr}/\Omega)\mathbf{v_p}$ and sum over all $\mathbf{p}$. We will again consider each term separately. The term originating from the collision integral gives the current fluctuations $\boldsymbol{\Delta}\mathbf{j}$. The remaining terms on the left-hand-side of Eq. (A4) require some more work.

The spatial gradient term yields the two terms in (22) which are associated with fluctuations in the diffusion currents. Performing the integral over the directions of $\mathbf{p}$, we have

$$\frac{e\tau_{\rm tr}}{\Omega}\sum_{\mathbf{p}}\mathbf{v_p}(\mathbf{v_p}\nabla_{\mathbf{r}}\Delta\tilde{n}_\epsilon) = \frac{e\tau_{\rm tr}}{m^*}N(0)\nabla_{\mathbf{r}}\int d\epsilon\,\epsilon\Delta\tilde{n}_\epsilon. \tag{A5}$$

From the definition

$$\Delta\tilde{n}_{\mathbf{p}} = \Delta n_{\mathbf{p}} - \frac{\partial n^0_{\mathbf{p}}}{\partial\epsilon_{\mathbf{p}}}\sum_{\mathbf{p}'}f_{\mathbf{pp}'}\Delta n_{\mathbf{p}'}, \tag{A6}$$

we find

$$\Delta\tilde{n}_\epsilon = \Delta n_\epsilon - \frac{\partial n^0_\epsilon}{\partial\epsilon}f_0\frac{\Omega}{e}\Delta\rho. \tag{A7}$$

Here, we introduced the angular average f_0 of the quasiparticle interaction function $f_{\mathbf{pp}'}$. The fluctuations of the distribution function occur in a narrow energy window around the Fermi energy. Hence, we can approximate

$$\Delta n_\epsilon = -A\,\frac{\partial n^0_\epsilon}{\partial\epsilon}, \tag{A8}$$

with

$$A = \frac{1}{eN(0)}\Delta\rho. \tag{A9}$$

Hence,

$$\Delta\tilde{n}_\epsilon = -\frac{1}{eN(0)}\frac{\partial n^0_\epsilon}{\partial\epsilon}(1+F_0)\Delta\rho. \tag{A10}$$

with the Landau parameter $F_0 = \Omega N(0) f_0$. Using this relation, we obtain for the spatial gradient term

$$\begin{aligned}
\frac{e\tau_{\rm tr}}{\Omega}\sum_{\mathbf{p}}\mathbf{v_p}(\mathbf{v_p}\nabla_{\mathbf{r}}\Delta\tilde{n}_\epsilon) &= \frac{\tau_{\rm tr}}{em^*N(0)}(1+F_0)\nabla_{\mathbf{r}}\{\rho\,\Delta\rho\} \\
&= D(1+F_0)\nabla_{\mathbf{r}}\Delta\rho + \Delta D(1+F_0)\nabla_{\mathbf{r}}\rho.
\end{aligned}\tag{A11}$$

with the diffusion-constant fluctuations given by $\Delta D = \tau_{\rm tr}\Delta\rho/em^*N(0)$.

The electric-field term becomes

$$\begin{aligned}
\frac{e^2\tau_{\rm tr}}{\Omega}\sum_{\mathbf{p}}\mathbf{v_p}(\mathbf{E}\nabla_{\mathbf{p}})\Delta n_\epsilon &= \frac{e^2\tau_{\rm tr}}{m^*}\mathbf{E}N(0)\int d\epsilon\,\epsilon\frac{\partial\Delta n_\epsilon}{\partial\epsilon} \\
&= -\frac{e\tau_{\rm tr}}{m^*}\Delta\rho\mathbf{E} \\
&= -\Delta\,\sigma\mathbf{E}.
\end{aligned}\tag{A12}$$

with $\Delta\sigma = e^2N(0)\Delta D$. By analogous steps, one finds that the term involving fluctuations in the electric field becomes

$$\frac{e^2\tau_{\rm tr}}{\Omega}\sum_{\mathbf{p}}\mathbf{v_p}(\boldsymbol{\Delta}\mathbf{E}\nabla_{\mathbf{p}})n_\epsilon = -\sigma\boldsymbol{\Delta}\mathbf{E}. \tag{A13}$$

This completes the derivation of the terms involving the electric field in (22).

The magnetic-field term is evaluated as

$$\begin{aligned}
&\frac{e\tau_{\rm tr}}{\Omega}e\sum_{\mathbf{p}}\mathbf{v_p}\left[(\mathbf{v_p}\times\mathbf{B})\nabla_{\mathbf{p}}(\mathbf{v_p}\boldsymbol{\Delta}\tilde{\mathbf{f}}_\epsilon)\right] \\
&\qquad = \frac{e^2\tau_{\rm tr}}{(m^*)^2\Omega}N(0)\int d\epsilon\,\epsilon(\mathbf{B}\times\boldsymbol{\Delta}\tilde{\mathbf{f}}_\epsilon) \\
&\qquad = \frac{e\tau_{\rm tr}}{m^*}(\mathbf{B}\times\boldsymbol{\Delta}\mathbf{j}),
\end{aligned}\tag{A14}$$

where I used

$$\begin{aligned}
\boldsymbol{\Delta}\mathbf{j} &= \frac{e}{\Omega}\sum_{\mathbf{p}}\mathbf{v_p}(\mathbf{v_p}\boldsymbol{\Delta}\tilde{\mathbf{f}}_\epsilon) \\
&= \frac{e}{m^*\Omega}N(0)\int d\epsilon\,\epsilon\boldsymbol{\Delta}\tilde{\mathbf{f}}_\epsilon.
\end{aligned}\tag{A15}$$

This completes the derivation of the response equation (22) of classical diffusive electrons in a magnetic field.

APPENDIX B: AVERAGE DISTRIBUTION FUNCTION

In this appendix, we provide details of the derivation of the diffusion equation (39) for the isotropic part of the

average distribution function from the Boltzmann equation (1). We decompose the average distribution function into its isotropic and anisotropic parts (in momentum $\mathbf{p}$),

$$n_{\mathbf{p}} = n_\epsilon + \mathbf{v_p f}_\epsilon \tag{B1}$$
$$J_{\mathbf{p}} = J_\epsilon + \mathbf{v_p J}_\epsilon. \tag{B2}$$

Inserting this decomposition into the Boltzmann equation (1) and collecting terms which are isotropic and anisotropic in the momentum $\mathbf{p}$, we obtain the two equations

$$(\mathbf{v_p}\nabla_\mathbf{r})\mathbf{v_p}\tilde{\mathbf{f}}_\epsilon - S_\mathbf{p}^{\mathrm{cf-cf}}(n_\mathbf{p}) = 0 \tag{B3}$$

and

$$(\mathbf{v_p}\nabla_\mathbf{r})\tilde{n}_\epsilon + e(\mathbf{E}+\mathbf{E}^{\mathrm{CS}})\mathbf{v_p}\frac{\partial n_\epsilon^0}{\partial\epsilon} + \frac{1}{\tau_{\mathrm{tr}}}\mathbf{v_p}\tilde{\mathbf{f}}_\epsilon = 0. \tag{B4}$$

Inserting the second into the first equation, using that the divergence of the Chern-Simons electric field vanishes, and expressing the physical electric field in terms of the scalar potential, one finds

$$D\nabla_\mathbf{r}^2\left\{\tilde{n}_\epsilon - e\phi\frac{\partial n_\epsilon^0}{\partial\epsilon}\right\} + S_\mathbf{p}^{\mathrm{cf-cf}}(n_\mathbf{p}) = 0. \tag{B5}$$

Using the relation[19]

$$\tilde{n}_\epsilon = n_\epsilon - \frac{\partial n_\epsilon^0}{\partial\epsilon}\sum_{\mathbf{p}'} f_{\mathbf{pp}'}\delta n_{\mathbf{p}'}, \tag{B6}$$

we obtain Eq. (39) within the accuracy of Fermi-liquid theory.

---

[1] J.K. Jain, Phys. Rev. Lett. **63**, 199 (1989); Phys. Rev. B **40**, 8079 (1989); Phys. Rev. B **41**, 7653 (1990); Adv. Phys. **41**, 105 (1992).
[2] B. Halperin, P.A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).
[3] For a review, see B. Halperin in *New perpectives in quantum Hall effects*," ed. by S. Das Sarma and A. Pinczuk (Wiley & Sons, 1997).
[4] A. Lopez and E. Fradkin, Phys. Rev. B **44**, 5246 (1991); Phys. Rev. B **47**, 7080 (1993).
[5] S.H. Simon and B.I. Halperin, Phys. Rev. B **48**, 17368 (1993).
[6] Y.B. Kim, A. Furusaki, X.-G. Wen, and P.A. Lee, Phys. Rev. B **50**, 17917 (1994).
[7] A. Stern and B.I. Halperin, Phys. Rev. B **52**, 5890 (1995).
[8] For a recent review of shot noise, see M.J.M. de Jong and C.W.J. Beenakker, Report No. cond-mat/9611140
[9] M. Reznikov, M. Heiblum, H. Shtrikman, and D. Mahalu, Phys. Rev. Lett. **75**, 3340 (1995); A. Kumar, L. Saminadayar, D.C. Glattli, Y. Yin, and B. Etienne, Phys. Rev. Lett. **76**, 2778 (1996).
[10] A.H. Steinbach, J.M. Martinis, and M.H. Devoret, Phys. Rev. Lett. **76**, 3806 (1996).
[11] G.B. Lesovik, JETP Lett. **49**, 592 (1989).
[12] C.W.J. Beenakker and M. Büttiker, Phys. Rev. B **46**, 1889(1992).
[13] K.E. Nagaev, Phys. Lett. A **169**, 103 (1992).
[14] B.L. Altshuler, L. Levitov, and A.Yu. Yakovets, JETP Lett. **59**, 857 (1994).
[15] K.E. Nagaev, Phys. Rev. B **52**, 4740 (1995).
[16] V.I. Kozub and A.M. Rudin, Phys. Rev. B **52**, 7853 (1995).
[17] Sh.M. Kogan and A.Ya. Shul'man, Sov. Phys. JETP **29**, 467 (1969).
[18] W. Kang, S. He, H.L. Stormer, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, Phys. Rev. Lett. **75**, 4106 (1995); it can be deduced from this experiment that $L_{\mathrm{cf-ph}}$ becomes orders of magnitude larger than ℓ_{tr} at $T \ll 1$K.
[19] D. Pines and P. Nozières, *The theory of quantum liquids*, (Addison-Wesley, 1989); we follow the notation of this book.
[20] Strictly speaking, the relaxation-time approximation is not justified for the magnetic scatterers which dominate for composite fermions [see e.g., A.D. Mirlin and P. Wölfle, Phys. Rev. Lett. **78**, 3717 (1997)]. However, one can show that a more realistic collision integral would not change our results, since we work in the diffusive regime.
[21] L.B. Ioffe, G.B. Lesovik, and A.J. Millis, Phys. Rev. Lett. **77**, 1584 (1996).
[22] C.L. Kane and M.P.A. Fisher, Phys. Rev. Lett. **72**, 724 (1994); P. Fendley, A.W.W. Ludwig, and H. Saleur, Phys. Rev. Lett. **75**, 2196 (1995).